COMPUTATIONAL MODELLING OF A SUBSONIC NONISENTROPIC FLOW OF A COMPRESSIBLE GAS IN THE DISCHRGE GAP OF A TRANSVERSE-FLOW CONTINUOUS WAVE LASER

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The need for computational modeling of a subsonic gas flow through a region of intensive energy release arose in particular because such a flow occurs in the discharge chamber of transverse-flow continuous wave (cw) CO_2 lasers [1, 2]. The distributions of the gasdynamic characteristics of the flow in the discharge zone are determined by the density Q of the power expended on heating the gas, which depends on the discharge power, the efficiency of excitation of laser levels, and the intensity of the output radiation. In most studies on the design of transverse-flow cw lasers [3-6], the flowin the discharge zone is modeled on the basis of a system of equations of one-dimensional gasdynamics. Such a quasi-two-dimensional model of the flow proposes that the gasdynamic characteristics be calculated from a set of horizontal sections of the flow, which altogether give a two-dimensional field of the distribution of the flow parameters in the discharge zone. In the general case the spatial distribution of the energy input Q can be rather nonuniform. This results in a nonzero transverse component of the flow velocity and, in the final account, a redistribution of the gasdynamic parameters between neighboring longitudinal sections. In principle, this cannot be taken into account in the quasi-two-dimensional approximation and, therefore, calculation in the one-dimensional model gives an unquestionably inaccurate result.

No studies have been carried out on the error of quasi-two-dimensional calculations in comparison with the two-dimensional gasdynamic approximation as well as on the dependence of this error on the flow parameters at the entrance to the discharge gap and on the absolute value of the energy input and its degree of nonuniformity.

Our aim here is to determine the limits of applicability of the model of one-dimensional gasdynamics for calculating the flow parameters in the discharge gap of a transverse-flow cw laser. For this purpose, on the basis of the solution of the system of gasdynamic equations of a nonviscous thermally nonconducting compressible gas, we studied the effects of the two-dimensionality of the subsonic steady-state flow in the discharge zone at flow parameters at the entrance of the discharge gap that are typical of most transverse-flow lasers.

1. Formulation of the Problem. We consider a plane steady-state flow of a compressible gas with initial velocity u_0 through the zone of energy release $\{0 \le x \le X,\ 0 \le y \le Y\}$ with an assigned field of power density Q. The occurs in a rectangular channel (Fig. 1).

We shall show that the viscosity and heat conduction can be disregarded in the model of the motion of the active medium of a transverse-flow cw laser with charactersitic velocity u_0 = 30 m/sec, pressure $p_0 = 2.67\cdot10^{-3}$ N/m, density $\rho_0 = 2\cdot10^{-2}$ kg/m^3 and an energy-release zone measuring $\ell_X = 0.1$ m and $\ell_Y = 0.05$ m. We assumed that the coefficient of dynamic viscosity of the mixture of gases in the active medium of a CO_2 laser is determined mainly by nitrogen: $\eta = 1.8\cdot10^{-5}$ kg/m·sec. Then the size the boundary layer is $\delta \sim \ell/\sqrt{\rho_0 u_0 \ell/\eta} \sim 10^{-3}$ m $\ll \ell_Y < \ell_X$. Thus, the flow in the discharge zone can be described in the model of a nonviscous gas, near the channel walls as well.

The characteristic scale associated with the heat-conducting mixture, which is determined primarily by helium (thermal conductivity coefficient λ_{He} = 0.3 W/m·K and heat capacity $C_p = 1\cdot10^{-3}$ cal/(kg·K), is

$$\Delta x_\lambda \sim \lambda_{He}/(\rho_0 u_0 C_p) \sim 10^{-4}\ \text{m} \ll l_y < l_x.$$

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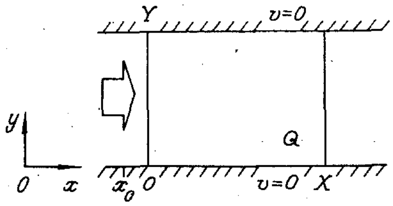


Fig. 1

Accordingly the steady-state distributions of the density ρ, longitudinal (u) and transverse (v) components of the velocity, and the pressure p in the discharge zone will be solutions of the system of gasdynamic equations

$$\frac{\partial}{\partial x}\mathbf{A}+\frac{\partial}{\partial y}\mathbf{B}=\mathbf{C},$$
$$\mathbf{A}=\begin{Bmatrix}\rho u\\ p+\rho u^2\\ \rho uv\\ \rho u(\varepsilon+p/\rho+\mathbf{v}^2/2)\end{Bmatrix},\tag{1.1}$$
$$\mathbf{B}=\begin{pmatrix}\rho v\\ \rho uv\\ p+\rho v^2\\ \rho v(\varepsilon+p/\rho+\mathbf{v}^2/2)\end{pmatrix},\quad \mathbf{C}=\begin{pmatrix}0\\0\\0\\Q\end{pmatrix}$$

[γ is the adiabatic exponent, $\mathbf{v}^2 = u^2 + v^2$, and ε is the internal energy, determined from the equation of state $\varepsilon = p/(\rho(\gamma - 1))$].

System (1.1) is supplemented withe the boundary conditions

$$\rho(0,\ y)=\rho_0,\quad u(0,\ y)=u_0,\ v(0,\ y)=0,\quad p(0,\ y)=p_0,$$
$$v(x,\ 0)=v(x,\ Y)=0.\tag{1.2}$$

The gas temperature is found from the equation $p = \rho RT/\mu$, where R is the universal gas constant and μ is the molecular mass of the mixture.

2. Method of Solution. At the subsonic flow velocitities characterististic of our problem system (1.1) is elliptical. Problems of this type are commonly solved by means of the steady-state method [7]. Instead of (1.1) we examine a nonstationary system of hyperbolic equations.

$$\frac{\partial}{\partial t}\mathbf{U}+\frac{\partial}{\partial x}\mathbf{A}+\frac{\partial}{\partial y}\mathbf{B}=\mathbf{C}\tag{2.1}$$

($\mathbf{U}=(\rho,\ \rho u,\ \rho v,\ \rho\varepsilon)$) with the initial conditions

$$\rho(x,\ y,\ 0)=\rho_0,\ u(x,\ y,\ 0)=u_0,\ v(x,\ y,\ 0)=0,$$
$$p(x,\ y,\ 0)=p_0.\tag{2.2}$$

As $t \to \infty$ the solution of (2.1), (2.2) tends to the solution of the stationary problem (1.1), (1.2), satisfying the laws of conservation of mass, momentum, and energy.

For the numerical calculation to find a steady-state solution approximating the solution of nonsteady problem (2.1), (2.2), we chose an explicit scheme for the decay of an discontinuity [7], which allows the flow parameters to be determined by a single method at points both in the interior points and on the boundary of the computing region.

Let us consider the formulation of the boundary conditions in greater detail. The conditions for no flow, i.e., $v_g = 0$, are assigned at the horizontal boundaries of the computing region. In the calculation of the decay of a discontinuity at these bounderies the oncoming wave is reflected from the well. A condition imposed at the right boundary is that no peturbations arrive into the computing region from the outside [8], which is also easily realized in the algorithm for calculating the decay of the discontinuity. In the initial stage of formation of a flow shock waves propagate from the front boundary of the region of energy release: downward and upward along the flow. The wave moving downward along the flow passes right through the right boundary without reflecting and, therefore, has no effect on the development of flow in the region of energy release. The other shock wave propagates without obstacle upward along the flow since its velocity is much lower than that of sound in the gas. Under the conditions without energy dissipation this wave can move upward along the flow to infinity, without being damped and leaving behind it some ρ^*, u^*, v^*, and p^*, which are independent of the magnitude of energy input. Thus, in order to formulate the conditions at the left boundary it is necessary to simulate the motion of a shock wave along an unperturbed flow to infinity. We propose to do this as follows. The left boundary is removed from the zone of energy release to a distance of three or four interlectrode gaps, where the oncoming shock wave can be assumed to have a plane front. The decay of the discontinuity is calculated at the boundary point x_0. To the left of x_0 the gas is assumed to be unperturbed, i.e., to have the initial flow parameters, white to the right it is assumed to have the parameters ρ^*, u^*, v^*, and p^* of the oncoming wave. The flow parameters established at point x_0 during the boundary decay of the discontinuity are taken to be boundary values. In this way the interaction of the shock wave with the initial flow is realized and as a result at the left boundary we determine $R_b = \rho(x_0)$, $U_b = u(x_0)$, and $P_b = p(x_0)$, which depend on both the parameter of the unperturbed flow and on the power of the energy release, thus reflecting the effect of the energy release on the subsonic flow of the gas ahead of the discharge region. As shown by calculations, this effect manifests itself in the lift effect of the flow, which is characteristic of the streamline flow problem. The longitudinal velocity U_b established at the left boundary, for example, is 15-20% lower than the initial flow velocity, in proportion to the input power. The new boundary values of the density and pressure differ little from the initial values:

$$(P_b - p_0)/p_0 \simeq 2\%, \; (R_b - \rho_0)/\rho_0 \simeq 2\ \%.$$

Stability difference schemes provide a choice of iterative step from time to time in accordance with the condition

$$\tau = F/(1/\tau_x + 1/\tau_y),$$

where $\tau_x = h_x/(c_0 + u_0)$; $\tau_y = h_y/(c_0 + v_0)$, F is the Courant number, c_0 is the velocity of sound in the unper turbed gas, and h_x and h_y are the spatial intervals of the net.

A major drawback of the steady-state method as applied to the problem formulated here is that it converges slowly to a stationary solution. This is due to the subsonic velocity of the flow in the entire region studied as well as the absence of dissipation processes in the mathematical model. The initial perturbations caused by the heating of the medium in the discharge zone propagate both downward and upward along the flow, interacting with each other and with the boundaries of the computing region. Since the model does include mechanisms of perturbation damping, such motions can continue to infinity without a steady flow being established. Since an approximation viscosity appears in the difference scheme, however, perturbation damping does occur and the flow becomes steady. In calculations carried out, about 2500 time steps were required for complete convergence; on a grid with 1200 nodes this took about 30 min on a BÉSM-6 computer.

Clearly, determination of the gasdynamic characteristics of the flow of the active medium of a laser on the basis of a solution of two-dimensional gasdynamic equations by the steady-state method is virtually inapplicable because of the long computer time needed. A more economical approach to the solution of system (1.1), (1.2), therefore, is required.

A characteristic feature of the steady-state solution of a nonstationary problem is an almost uniform pressure field in the discharge zone (e.g., the maximum deviation from pressure uniformity is less than 0.5%). Evidently, the approximation of a constant pressure p(x, y) = const should be considered to be good enough for solving the stationary system (1.1) (1.2). In the momentum transfer equation, however, the velocity components are determined by the pressure gradient

$$(\mathbf{v}\cdot\nabla)\mathbf{v} = 1/\rho\cdot\nabla p. \quad (2.3)$$

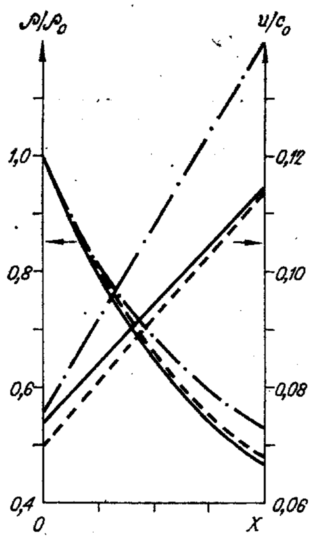


Fig. 2

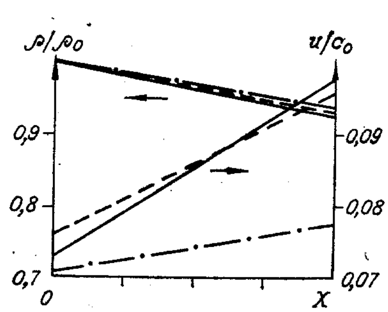


Fig. 3

Satisfactory information about u and v cannot be obtained from (2.3) at p = const. This difficulty can be obviated by going over to the new variables ψ and ω which are derived as follows:

$$\rho u = \partial\psi/\partial y,\ \rho v = -\partial\psi/\partial x,\ \omega = \mathrm{curl}\,\mathbf{v} = \partial v/\partial x - \partial u/\partial y. \qquad (2.4)$$

The equations of steady-state gasdynamics (1.1) in these variables are transformed to the form [9, 10]

$$\mathrm{div}(\rho\mathbf{v}\omega) = -\mathrm{curl}\,(\rho\cdot\nabla v^2/2),\quad \mathrm{div}(\rho\mathbf{v}H) = Q \qquad (2.5)$$

($H = w + v^2/2$ and $w = \gamma/(\gamma-1)\cdot p/\rho$ is the enthalpy of the gas). Setting $p = p_0$ = const, we close system (2.4), (2.5). It can be solved by the iteration method (for more details on the numerical algorihm of this method see [9, 10]).

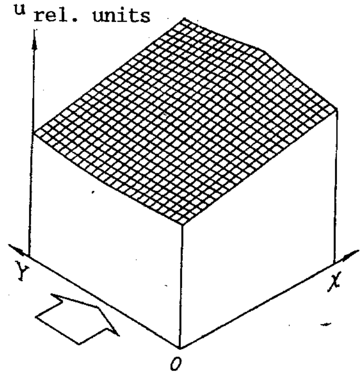
As shown by calculations, the approximation of a constant pressure $p(x, y) = p_0$ is sufficiently reliable for the problem under consideration. This follows from the good agreement between the results of solving the gasdynamic equations by the steady method and the equations in the variables ψ and ω by the iteration method. We note that implementation of the iteration method of solving two-dimensional equations meant an almost twenty-fold saving of time in comparison with solution by the iteration method.

3. Results of Calculations and Discussion. We carried out a series of model computations of the distributions of the gas-flow parameters in the discharge zone with different configurations of the region of energy release and distributions of the density of the energy input Q in the volume. The flow parameters at the entrance into the discharge zone were u_0 = 30 m/sec, $p_0 = 2.67\cdot10^3$ N/m², T_0 = 300 K, which is characteristic of most transverse-flow lasers.

Suppose that the distribution of the discharge-power distribution in the zone of energy release is

$$Q(x,y) = \begin{cases} Q_0 y/(0.5Y), & 0 \leqslant x \leqslant X,\ 0 \leqslant y \leqslant 0.5Y, \\ Q_0 (Y-y)/(0.5Y), & 0 \leqslant x \leqslant X,\ 0.5Y \leqslant y \leqslant Y, \end{cases} \qquad (3.1)$$

where Q_0 was chosen so as to ensure a roughly two-fold rise in temperature at the end of the discharge zone along layer y = 0.5 Y. The distributions of the normalized longitudinal velocity u/c_0 ($c_0 = \sqrt{\gamma R T_0}$ is the velocity of sound) and of the gas density ρ/ρ_0 along the flow with a given model field of energy input (3.1) are shown in Figs. 2 and 3. The distributions along the horizontal section y = 0.5 Y, where the energy input is maximum, are shown in Fig. 2 and along the section y = 0 (Q = 0), in Fig. 3. The dashed curve corresponds to the solution of two-dimensional nonstationary gasdynamic equations by the steady-state method and the solid curve corresponds to the solution of the equations in the variables ψ and ω by the interation method in the constant-pressure approximation. The results of the solution of the two-



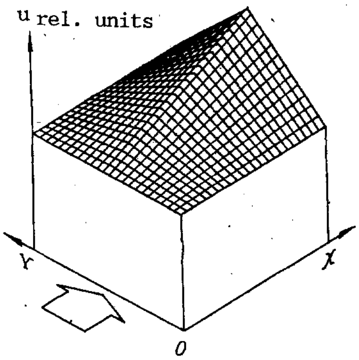


Fig. 4 Fig. 5

dimensional equations by the indicated methods are in good agreemnt. The largest deviation in the solutions for the longitudinal velocity is δu = 5% at the beginning of the energy-input zone; this apparently is attributable to the lift effect of the flow, which is most pronounced in the solution of the nonstationary problem. The solutions for the density and temperature have smaller discrepancies, the largest being $\delta\rho \simeq 3\%$ and $\delta T \simeq 3\%$ at the right boundary of the discharge zone.

The dash-dot lines in Figs. 2 and 3 represent the quasi-two-dimensional solution of the gasdynamic equations without allowance for the variation of the gasdynamic paramters in the direction transverse to the flow. A comparison of the quasi-two-dimensional and two-dimensional solution reveals that the largest deviations are observed in the solutions for the longitudinal flow velocity. For the energy-input distribution considered, with a maximum in the central horizontal layer, when the transverse velocity components are included in the two-dimensional model the gas flows more quickly along the boundary layers, where the energy input is minimum, and, conversely, slows down along the layer of maximum energy input. As a result, at the right boundary of the discharge zone the two-dimensional model underestimates the longitudinal velocity by almost 20% in the central layer and overestimates it by 30% at the walls in comparison with the quasi-two-dimensional solution.

Figures 4 and 5 show the distribution fields of the longitudinal flow velocity in the discharge zone, which were obtained, respectively, in the two-dimensional and quasi-dimensional gasdynamic solutions. On the whole, the pattern of flow in the two-dimensional solution is qualitatively different than in the quasi-two-dimensional solution. Because of the lift effect of the flow near the left boundary the gas flows more quickly at the walls than at the center, where the energy release is high. Further down the flow the gas in the central layers of the flow is accelerated more quickly and as a result the velocity profile is evened out in the cross sections near the middle of the discharge zone. At the right boundary the velocity at the wall is already 90% of the velocity in the central layer (50% in the quasi-two-diemnsional solution). Thus, a smoother profile of the longitudinal velocity in sections transverse to the flow is characteristic of the two-dimensional solution.

The transverse components of the flow-velocity vector in the case under consideration were small, amounting to no more than 7-8% of the longitudinal velocity components.

In regard to the density and temperature the two-dimensionality effect manifests itself only in layer of maximum energy input. Here at the end of the discharge the density decreases and the temperature rises by 10% in comparison with the quasi-two-dimensional solution. This is because in the two-dimensional solution the gas flows more slowly in the central layers, remains in the discharge zone longer, and hence heat up more. The hotter gas is expelled from the central layer of the flow to the walls of the channel. It would seem that the density in the boundary layers of the flow should increase in comparison with the quasi-two-dimensional solution. This does not happen, however, because the gas moves more quickly near the walls than in the quasi-two-dimensional case and the main mass transport occurs in the boundary regions. The mass flow at the walls in the two-dimensional solution is 1.8 times that at the center. As a result of a rise in gas density and a corresponding decrease in gas temperature near the walls, where the energy input is a minimum, this does not occur.

When the transverse velocity components are included in the two-dimensional model the mass flow $m = \rho u$, with a uniform field $m(x, y) = \rho_0 u_0$ in the quasi-two-dimensional solution,

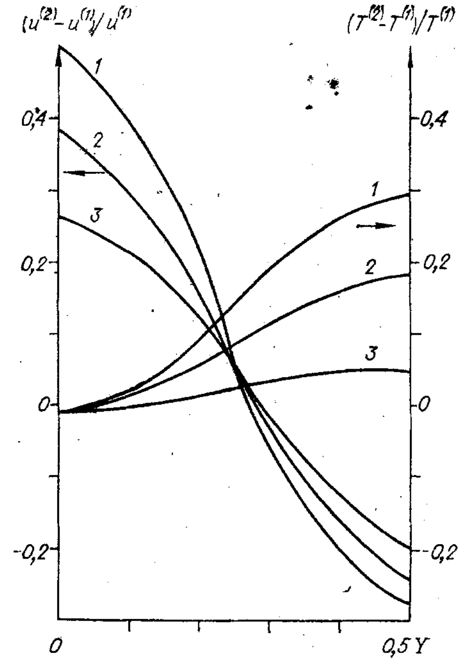


Fig. 6

is redistributed substantially among the horizontal sections. Near the channel walls at the end of the discharge zone the mass flow in the two-dimensional solution is 1.3 times that in the quasi-two-dimensional solution and decreases by a factor of 1.4 with respect to the central layer of maximum energy input. The gas flow rate through the cross section x = X remains constant in both cases.

Let us consider how much the solutions deviate when the initial flow velocity is varied. Figure 6 shows the relative deviation of the two-dimensional solution for velocity $u^{(2)}$ and temperature $T^{(2)}$ from the quasi-two-dimensional solution $u^{(1)}$ and $T^{(1)}$ in the cross section x = X of the flow at the end of the discharge zone, which was obtained for a Gaussian distribution of the input-power density in the discharge zone:

$$Q(x, y) = Q_0 \exp\{-0.01 \times \times [(x - x^*)^2 + (y - y^*)^2]\}$$
$$(Q_0 = 10 \text{ W/cm}^2,\ x^* = 0.5X,\ y^* = 0.5\,Y).$$

The curves correspond to different values of the initial flow velocity: 1-3) u_0 = 20, 40, 70 m/sec at $p_0 = 2.67 \cdot 10^3$ N/m², T_0 = 300 K. Into our discussion we introduce the quantity $k_T = \max T(x, y)/T_0$, which characterizes the heating of the gas. For the given case k_T = 3.5, 2, and 1.5.

From the graphs we see that the largest deviation of the solutions for the velocity are observed in regions of extreme energy release and decrease as the initial flow velocity rises, which corresponds to a decrease in k_T. In this case there exists a horizontal layer, near the layer of half energy release, along which the solutions for the velocity coincide, regardless of the initial flow velocity. The deviation of the solutions for the gas temperature also decrease with decreasing initial velocity. Near the walls, where the energy input is minimum, the solutions virtually coincide, regardless of the initial velocity.

In summary, for any distribution of the power input there exist such initial flow velocities that the difference between the solutions are small and the quasi-two-dimensional model can be used to find the flow parameters in the zone of energy input with good accuracy. For the case under consideration this is possible at flow velocities $u_0 \geq 70$ m/sec. The heating of the gas depends on both how long the gas is in the discharge zone, i.e., the flow velocity, and on the absolute value of the energy input Q. In this sense the temperature distribution in the discharge zone is the result of a distribution of the longitudinal flow velocity in the the discharge zone. The applicability of the quasi-two-dimensional model to the calculation of the flow characteristics must be assessed from the deviation in the solution for the temperature and we can consider the heating k_T of the gas to be the most universal criterion of applicability of the quasi-two-dimensional model. Accordingly, the following approach is proposed for selecting the model for calculating the gasdynamic characteristics of the flow of active medium in a transverse-flow gas-discharge laser. If the degree to which the gas is heated by the end of the discharge zone does not exceed k_T = 1.5-1.7, then the quasi-two-dimensional model can be used to find the flow parameters with good accuracy ($\delta T/T \leq 5\%$, which is comparable with the experimental error).

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